SINGULAR INTEGERS IN CYCLOTOMIC FIELDS

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After proving Fermat's last theorem for regular primes, Ernest Kummer tried to extend his method to include singular primes. He showed that if the first case fails for a prime p, then p must divide the $(p-3)$-rd and $(p-5)$-th Bernouilli numbers. For this he used the Kummer-Stickelberger theorem on annihilators of the class group of a cyclotomic field. His methods can be used to gain more information about singular integers in cyclotomic fields. An algebraic integer in the p-th cyclotomic field is called singular if the principal ideal it generates is the p-th power of some other ideal. For these singular integers one can use the same techniques as Kummer used, see for example the PhD-thesis of Andrew Granville. The case of Fermat's last theorem then corresponds to a singular integer of the form $x + y\zeta$, where ζ is a primitive p-th root of unity. In this talk we shall focus on the case of a singular integer of the form $x + y\zeta + z\zeta^2$. First we see how much information we obtain with Kummer's method. Then we shall use a different method involving the p-adic logarithm to gain additional information. Combining both methods we obtain a 'first case' for these singular integers if p does not divide the $(p-3)$-rd Bernouilli number.

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