# COMPLEX HAMILTONIAN DYNAMICS AND GEODESICS IN THE SPACE OF KÄHLER METRICS

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Complex dynamics is an extension of the usual Hamiltonian evolution, in which the Hamiltonian function and the time parameter are considered to be complex valued. The purpose of this talk is to explore the connections between this extension and the geometry and analysis of Kähler manifolds [Don99], by studying specific examples of complex evolution.

We start by presenting a short overview of the formalism used in [MN15] to give meaning to the notion of complex dynamics on a complex manifold $(M, J)$. We then introduce a Kähler strucutre $(\omega, J, g)$ on $M$ and study how this structure changes with the complex flow. After that we consider the geodesic problem (for the Mabuchi metric) on the infinite-dimensional manifold of Kähler forms with fixed cohomology class on $M$. We show that the analytic solutions of the geodesic equation can be obtained as an imaginary time flow of a real-analytic hamiltonian vector field.

We finish by illustrating the results with two new examples. Firstly, for $M = \mathbb{R}^2$, we consider the hamiltonian of a harmonic oscillator, where a fraction of its kinetic energy is imaginary. Secondly, for $M = S^2$, we consider the imaginary time evolution for the hamiltonian that is half the square of the moment map for an $S^1$-action on $S^2$. In both cases, we study the evolution of the Kähler structure under the complex flow, and use graphical representations whenever possible.

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## References

[Don99] S. K. Donaldson. Symmetric spaces, Kähler geometry and Hamiltonian dynamics. In *Northern California Symplectic Geometry Seminar*, volume 196 of *Amer. Math. Soc. Transl. Ser. 2*, pages 13–33. Amer. Math. Soc., Providence, RI, 1999.

[MN15] José M. Mourão and João P. Nunes. On complexified analytic Hamiltonian flows and geodesics on the space of Kähler metrics. *Int. Math. Res. Not. IMRN*, (20):10624–10656, 2015.

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