SUPERCLOSENESS-SUPRACONVERGENCE IN SCHEMES FOR DRUG DELIVERY ENCHANCED BY ULTRASOUND

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In this presentation we study a system of a hyperbolic equation and two parabolic equations. This system can be used to describe drug transport enhanced by ultrasound. In this case, the hyperbolic equation describes the ultrasound propagation through the target tissue. The heat propagation generated by the ultrasound and the drug transport are described by the mentioned parabolic equations.

The ultrasound propagation leads to an increase in the temperature of the system. Therefore the reaction term of the equation for the temperature depends on the solution of the hyperbolic equation. The ultrasound also induces a convective transport and structural changes that increase drug transport. Then the convective and diffusion coefficients of the parabolic equation for drug concentration depend on the solution of the hyperbolic equation. The drug diffusion coefficient also depends on the temperature.

From numerical point of view, we propose a numerical method for such system that can be seen simultaneously as a FDM and a fully discrete piecewise linear FEM. We prove second order of convergence with respect to a discrete H^1-norm.

References

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