Summer school 2018

Book of Abstracts
(contributed talks)

University of Aveiro, Department of Mathematics
September 3 – 7 and 10 – 14

Jointly organised by the PhD program in Mathematics of the University of Aveiro, the PhD program map-pdma in Applied Mathematics of the Universities of Aveiro, Minho and Porto, and the joint PhD program in Mathematics of the Universities of Coimbra and Porto.

Contents

Diffusion in Multispecies Gas Mixtures

Benjamin Anwasia
University of Minho, Braga, Portugal
id6226@alunos.uminho.pt

The Maxwell-Stefan equations are often used to describe non-typical diffusion which arises as a consequence of thermodynamic non-idealities. Starting from the kinetic theory, we will show how to formally derive, in the hydrodynamic limit, the Maxwell-Stefan law for reactive and non- reactive multispecies gas mixtures.

References

[1] J. B. Duncan, H. L. Toor. *An experimental study of three component gas diffusion*, AIChE Journal, 8(1):38–41, 1962.

[2] R. Krishna, J. Wesselingh. *The Maxwell-Stefan approach to mass transfer*, Chem. Engin. Sci., 52, 861–911, 1997.

[3] B. Anwasia, P. Gonçalves, A. J. Soares. *From the simple reacting sphere kinetic model to the reaction-diffusion system of Maxwell-Stefan type*, preprint arXiv:1707.01316, 2017

Reactive transition models to biological systems

Daniel Figueiredo and Manuel A. Martins
CIDMA – University of Aveiro, Portugal
daniel.figueiredo@ua.pt; martins@ua.pt

Reactive frames have been study by Gabbay and his collaborators (see [1]) and are those frames whose structure is not fixed,*i.e.*, can vary according to the path chosen. Informally, a reactive frame can be seen as a digraph (V, E) where the set of edges can be altered whenever an edge is crossed. Some particular cases of these kind of structures have been proposed by authors like Areces and van Benthem (see [2], [3]) that proposed swap and sabotage logics.

We used this kind of model to describe biochemical events which usually are not naturally modeled or that require the consideration of additional components in order to be correctly described. In particular, we show that this approach can lead to more intuitive models for many biochemical systems. In particular, we focus our attention in biological regulatory networks.

We conclude with some ideas about how fuzziness/probability can be introduced in these models and how it can be useful to describe biological phenomena. In particular, we present some examples of how this would more detailedly describe the referred biological regulatory networks.

References

[1] D. M. Gabbay, S. Marcelino, (2009). Modal logics of reactive frames. *Studia Logica* 93(2), pp. 405-446

[2] J. van Benthem (2005). An essay on sabotage and obstruction. In: *Mechanizing Mathematical Reasoning*, pp. 268–276. Springer

[3] C. Areces, R. Fervari, G. Hoffmann (2015). Relation-changing modal operators. *Logic Journal of IGPL* p. jzv020

[4] P. Blackburn, M. De Rijke, Y. Venema (2002). *Modal Logic: Graph. Darst*, vol. 53. Cambridge University Press

[5] H. De Jong (2002) Modeling and Simulation of Genetic Regulatory Systems - A Literature Review. *Journal of computational biology*, 9.1: 67–103.

Randić index and Randić matrix: properties and some applications

Helena Gomes
(joint work with Enide Andrade and Maria Robbiano)
CIDMA UA|School of Education of Viseu|Polytechnic Institute of Viseu
hgomes@ua.pt|hgomes@esev.ipv.pt

The Randić index is a spectral invariant presented in 1975 by Milan Randić and with important applications in Chemistry, Pharmacology and Medicine namely, in the study of prediction of colon and breast cancer. In 2010 the Randić matrix was defined as a nonnegative matrix built from this index. The study of the spectrum of matrices associated with graphs is one of the major goals of research in graph theory and there are already several applications in different scientific areas. This work presents a study related to spectral invariants for the Randić matrix of a graph namely the Randić spread (new concept in literature) and in addition, upper and lower bounds are presented for this spectral invariant. In chemistry, the energy of caterpillar graphs, that are associated with aromatic systems, is related with the resonance of these systems. Having this as motivation, the spectrum and the Randić spectrum of caterpillar graphs are studied and upper bounds are presented for the energy and for the Randić energy of this class of graphs.

References

[1] Andrade, E., Gomes, H. and Robbiano, M. Spectra and Randić Spectra of Caterpillar Graphs and Applications to the Energy. *MATCH Commun. Math. Comput. Chem.*, 77:61-75, 2017.

[2] Bozkurt, Ş. B., Güngör, A. D. and Gutman, I. Randić spectral radius and Randić energy. *MATCH Commun. Math. Comput. Chem.*, 64:321–334, 2010.

[3] Brauer, A. Limits for the characteristic roots of a matrix. IV. Applications to stochastics matrices. *Duke Math. J.*, 19:75–91, 1952.

[4] Cvetković, D., Doob, M., Gutman, I. and Torgasev, A. *Recent Results in the Theory of Graph Spectra.* North-Holland, Amsterdam, 1988.

[5] Gomes, H., Gutman, I., Martins, E., Robbiano, M. and San Martín, B. On Randić Spread. *MATCH Commun. Math. Comput. Chem.*, 72:249-266, 2014.

[6] Gomes, H., Martins, E. A., Robbiano, M. and San Martín, B. Upper bounds for Randić spread. *MATCH Commun. Math. Comput. Chem.*, 72:267-278, 2014.

Indecomposable representations of left regular bands

Herman Goulet-Ouellet
Université du Québec à Montréal, Montréal (Québec), Canada
goulet-ouellet.herman@courrier.uqam.ca

Left regular bands are a family of semigroups that enjoy many surprising properties. They have received a lot of attention recently due to their link with some Markov chains (see [1]). In particular, a lot of informations about random walks on the chambers of an hyperplan arrangement can be computed using the representation theory of left regular bands.

In this talk, we will show how to compute the indecomposable projective representations of left regular bands. The first part will introduce the basic definitions and tools needed. The second part will present both a recursive construction (due to Saliola [2]) and a closed formula, used to compute complete sets of primitive orthogonal idempotents of left regular bands algebras. These sets, called eulerian families, correspond to the indecomposable projective representations of left regular bands over arbitrary fields. In the third part we will show, using a method introduced in [3], how these eulerian families can be used to study certain Markov chains.

References

[1] Brown, Kenneth S. Semigroup and ring theoretical methods in probability *Representations of finite dimensional algebras and related topics in Lie theory and geometry*, 3–26, Fields Inst. Commun., 40, Amer. Math. Soc., Providence, RI, 2004.

[2] Saliola, Franco V. The Quiver of the Semigroup Algebra of a Left Regular Band *Int. J. Algebra Comput.*, 17, 1593, 2007.

[3] Saliola, Franco V. Eigenvectors for a random walk on left-regular bands *Adv. in Appl. Math.*, 48(2), 206-311, 2012.

On Wiener's Tauberian theorems and convolution for oscillatory integral operators

Rita Guerra
CIDMA, Department of Mathematics, University of Aveiro
ritaguerra@ua.pt

The main aim of this talk is to present new Paley-Wiener and Wiener's Tauberian results associated with an oscillatory integral operator (which is depending on cosine and sine kernels), as well as to introduce a consequent new convolution. Additionally, a new Young-type inequality for the obtained convolution is proved, and a new Wiener type algebra is also associated with this convolution.

References

[1] Brascamp, Herm Jam; Lieb, Elliot. Best constants in Young's inequality, its converse, and its generalization to more than three functions. *Adv Math* 20: 151–173, 1976.

[2] Castro, Luís Pinheiro; Guerra, Rita Correia; Tuan, Nguyen Minh. Heisenberg uncertainty principles for an oscillatory integral operator. *American Institute of Physics, AIP Proc* 1798(1): 020037, 10pp., 2017.

[3] Frank, Rupert; Sabin, Julien. Spectral cluster bounds for orthonormal systems and oscillatory integral operators in Schatten spaces. *Adv Math* 317: 157–192, 2017.

[4] Korevaar, Jacob. A century of complex Tauberian theory. *Bull Amer Math Soc* (NS) 39: 47–531, 2002.

[5] Korevaar, Jacob. *Tauberian Theory: A Century of Developments.* Grundl math Wiss 329, Springer-Verlag, 2004.

[6] Korevaar, Jacob. Distributional Wiener-Ikehara theorem and twin primes. *Indagationes Mathematicae* (NS) 16: 3–49, 2005.

[7] Motygin, Oleg Valerievich. Numerical approximation of oscillatory integrals of the linear ship wave theory. *Appl Numer Math* 115: 99–113, 2017.

[8] Novak, Erich; Ullrich, Mario; Woźniakowski, Henryk; Zhang, Shun. Complexity of oscillatory integrals on the real line. *Adv Comput Math* 43(3): 537–553, 2017.

[9] Pitt, Harry Raymond. General Tauberian theorems. *Proc London Math Soc* 44: 243–288, 1938.

[10] Rudin, Walter. Functional Analysis. McGraw-Hill, 1991.

[11] Sjölin, Per. Some remarks on singular oscillatory integrals and convolution operators. *Proc Am Math Soc* 145(9): 3843–3848, 2017.

[12] Tuan, Nguyen Minh; Tuan, Phan Duc. Operator properties and Heisenberg uncertainty principles for a un-unitary integral operator. *Integral Transforms Spec Funct* 23: 1–12, 2012.

[13] Veys, Willem; Zúñiga-Galindo, Wilson. Zeta functions and oscillatory integrals for meromorphic functions. *Adv Math* 311: 295–337, 2017.

[14] Xiao, Lechao. Endpoint estimates for one-dimensional oscillatory integral operators. *Adv Math* 316: 255–291, 2017.

[15] Xu, Zhenhua; Xiang, Shuhuang. Gauss-type quadrature for highly oscillatory integrals with algebraic singularities and applications. *Int J Comput Math* 94(6): 1123–1137, 2017.

Supercloseness-Supraconvergence in Schemes for Drug Delivery Enchanced by Ultrasound

J. A. Ferreira, D. Jordão, L. Pinto

CMUC, Departamento de Matemática, Universidade de Coimbra, Apartado 3008, EC Santa Cruz, 3001-501 Coimbra, Portugal

ferreira@mat.uc.pt, djordao@mat.uc.pt, luisp@mat.uc.pt

In this presentation we study a system of a hyperbolic equation and two parabolic equations. This system can be used to describe drug transport enhanced by ultrasound. In this case, the hyperbolic equation describes the ultrasound propagation through the target tissue. The heat propagation generated by the ultrasound and the drug transport are described by the mentioned parabolic equations.

The ultrasound propagation leads to an increase in the temperature of the system. Therefore the reaction term of the equation for the temperature depends on the solution of the hyperbolic equation. The ultrasound also induces a convective transport and structural changes that increase drug transport. Then the convective and diffusion coefficients of the parabolic equation for drug concentration depend on the solution of the hyperbolic equation. The drug diffusion coefficient also depends on the temperature.

From numerical point of view, we propose a numerical method for such system that can be seen simultaneously as a FDM and a fully discrete piecewise linear FEM. We prove second order of convergence with respect to a discrete H^1-norm.

References

[1] Couture, Olivier et al. Review of ultrasound mediated drug delivery for cancer treatment: updates from pre-clinical studies. *Translational Cancer Research* 3(5):494-511, 2014.

[2] Ferreira, J. A., and Grigorieff, R. D. Supraconvergence and supercloseness of a scheme for elliptic equations on nonuniform grids. *Numerical Functional Analysis and Optimization*, 27(5-6):539–564, 2006.

[3] Ferreira, J. A., Jordão, D., and Pinto, L. Approximating coupled hyperbolic-parabolic systems arising in enhanced drug delivery. *Computers and Mathematics with Applications*, 76:81-97, 2018.

[4] Ferreira, J. A., Jordão, D., and Pinto, L. Second order approximations for kinetic and potential energies in Maxwell's wave equations. *Applied Numerical Mathematics*, 120:125-140, 2017.

[5] Pulkkinen, Aki et al. Numerical simulations of clinical focused ultrasound functional neurosurgery. *Physics in Medicine & Biology* 59: 1679–1700, 2014.

Epidemiological modelling and optimal control of a financial-virus spreading

Olena Kostylenko[1]

Center for Research and Development in Mathematics and Applications (CIDMA),
Department of Mathematics, University of Aveiro, Aveiro, 3810-193, Portugal

o.kostylenko@ua.pt

The global crisis of 2008 threatened the stability of the world economy, the negative consequences of it can be felt even today. This makes the subject an interesting field of research. The spread of an infection among a certain population is very similar to the contagion process in economy. This similarity allows us to consider the contagion in the global economy using the same mathematical models for the spread of a disease, which are used in epidemiology. Our research focuses on the dynamic behaviour of financial viruses that spread through countries' network interconnections. To avoid significant financial losses, an optimal control problem is formulated. The proposed approach describes well the reality of the world economy, the importance of countries' interconnectedness and recommendations are also made to minimize risks in the global system.

[1]Supported by CIDMA, project UID/MAT/04106/2013, and FCT Ph.D. fellowship PD/BD/114188/2016, under supervision of D. F. M. Torres and H. S. Rodrigues.

An epidemic model with vaccination for the worst cholera outbreak in the history

Ana P. Lemos-Paião, Cristiana J. Silva and Delfim F. M. Torres

Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

anapaiao@ua.pt

We revisit the cholera mathematical model analysed in [1], adding a vaccination class. We show that the proposed model is epidemiologically and mathematically well posed and it only has two equilibrium points: a disease-free and an endemic. The basic reproduction number is determined and the local asymptotic stability of equilibria is studied. The worst cholera outbreak began on 27th April 2017 and it has occurred in Yemen. Between 27th April 2017 and 15th April 2018 there were 2275 deaths due to this epidemic [3]. A vaccination campaign began on 6th May 2018 and ended on 15th May 2018 [2]. Through numerical simulations, we show that the model fits well this outbreak and observe that if a vaccination campaign had begun earlier the number of infected individuals would have been significantly lower.

References

[1] Lemos-Paião, A. P., Silva, C. J. and Torres, D. F. M. An epidemic model for cholera with optimal control treatment. *J. Comput. Appl. Math.*, 318:168–180, 2017.

[2] World Health Organization, Yemen crisis: Fighting the world's largest cholera outbreak: oral cholera vaccination campaign begins in Yemen, 06th June 2018, http://www.emro.who.int/pdf/yem/yemen-news/oral-cholera-vaccination-campaign-in-yemen-begins.pdf?ua=1.

[3] World Health Organization, Yemen: Weekly Epidemiological Bulletin W15 2018, 21st May 2018, http://www.emro.who.int/images/stories/yemen/week_15.pdf?ua=1.

Singular Integers in Cyclotomic Fields

Peter Lombaers
Univsersidade do Porto, Mathematics Department
p.lombaers@gmail.com

After proving Fermat's last theorem for regular primes, Ernest Kummer tried to extend his method to include singular primes. He showed that if the first case fails for a prime p, then p must divide the $(p-3)$-rd and $(p-5)$-th Bernouilli numbers. For this he used the Kummer-Stickelberger theorem on annihilators of the class group of a cyclotomic field. His methods can be used to gain more information about singular integers in cyclotomic fields. An algebraic integer in the p-th cyclotomic field is called singular if the principal ideal it generates is the p-th power of some other ideal. For these singular integers one can use the same techniques as Kummer used, see for example the PhD-thesis of Andrew Granville. The case of Fermat's last theorem then corresponds to a singular integer of the form $x + y\zeta$, where ζ is a primitive p-th root of unity. In this talk we shall focus on the case of a singular integer of the form $x + y\zeta + z\zeta^2$. First we see how much information we obtain with Kummer's method. Then we shall use a different method involving the p-adic logarithm to gain additional information. Combining both methods we obtain a 'first case' for these singular integers if p does not divide the $(p-3)$-rd Bernouilli number.

Generalized Heisenberg Algebras and their Poisson semiclassical limit

Farrokh Razavinia
Universidade do Porto
up201703457@fc.up.pt

This talk will be about the Weyl algebras and some of their important structures and also the generalized Weyl algebras, generalized Heisenberg algebras and their relations and some of their important examples and structures (their Poisson structures). At the end we will try to have a short introduction to the Dixmier and the Jacobian conjecture and their eqivalence and if time allows, we will have a walk through, for to get in the Belov-Kontsevich conjecture about the automorphism group of the n-th Weyl algebra.

References

[1] Belov-Kanel, Alexei, and Maxim Kontsevich. *The Jacobian conjecture is stably equivalent to the Dixmier conjecture.* Mosc. Math. J 7.2 (2007): 209-218.

[2] Belov-Kanel, Alexei, and Maxim Kontsevich. *Automorphisms of the Weyl algebra.* Letters in mathematical physics 74.2 (2005): 181-199.

[3] Kanel-Belov, Alexei, Andrey Elishev, and Jie-Tai Yu. *Automorphisms of Weyl Algebra and a Conjecture of Kontsevich.* arXiv preprint arXiv:1802.01225 (2018).

[4] Vladimir V. Bavula. *Generalized Weyl algebras and their representations.* St. Petersburg Mathematical Journal 4 (1993), no. 1, 71-92.

[5] Rencai Lü and Kaiming Zhao. *Finite-dimensional simple modules over generalized Heisenberg algebras.* Linear Algebra Appl., 475:276–291, 2015.

[6] Samuel A. Lopes. *Non-Noetherian generalized Heisenberg algebras.* J. Algebra Appl., 16(2):1750064, 2017.

[7] Van den Essen, Arno. *Polynomial Automorphisms: and the Jacobian Conjecture.* Vol. 190. Birkhäuser, 2012.

Dealing with urcertainty in the Maritime Inventory Routing Problem

Filipe Rodrigues
University of Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal
fmgrodrigues@ua.pt

Maritime transportation is one of the most common ways to transport goods. However, this kind of transportation is characterized by high levels of uncertainty, since the weather conditions have a great impact in the sailing times.

We consider a single product maritime inventory routing problem in which the production and consumption rates are constant over the planning horizon. The problem involves a heterogeneous fleet and multiple production and consumption ports with limited storage capacity.

The impact of the uncertainty in the sailing times is analysed according to five different approaches: deterministic model, deterministic model with safety stocks, robust optimization, stochastic programming, and a stochastic model using the conditional value at risk measure. This last approach has never been used to solve this problem. The non-deterministic approaches assume two-stage decisions, where the routing as well as the quantities to load/unload are fixed before the uncertainty is revealed, while the visit time to ports and the inventory levels are adjusted to the scenario. For each approach, a mathematical formulation is proposed. An extensive computational comparison of the proposed approaches, based on several quality parameters, is carried out over a set of 21 instances. The obtained results show that substantial gains can be obtained when the uncertainty is incorporated into the model.

Complex Hamiltonian dynamics and geodesics in the space of Kähler metrics

Pedro Miguel Silva

Departament of Mathematics, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa
Departament of Mathematics, Universidade de Coimbra, 3001-454 Coimbra

`pedromiguelsilva@tecnico.ulisboa.pt`

Complex dynamics is an extension of the usual Hamiltonian evolution, in which the Hamiltonian function and the time parameter are considered to be complex valued. The purpose of this talk is to explore the connections between this extension and the geometry and analysis of Kähler manifolds [Don99], by studying specific examples of complex evolution.

We start by presenting a short overview of the formalism used in [MN15] to give meaning to the notion of complex dynamics on a complex manifold (M, J). We then introduce a Kähler strucutre (ω, J, g) on M and study how this structure changes with the complex flow. After that we consider the geodesic problem (for the Mabuchi metric) on the infinite-dimensional manifold of Kähler forms with fixed cohomology class on M. We show that the analytic solutions of the geodesic equation can be obtained as an imaginary time flow of a real-analytic hamiltonian vector field.

We finish by illustrating the results with two new examples. Firstly, for $M = \mathbb{R}^2$, we consider the hamiltonian of a harmonic oscillator, where a fraction of its kinetic energy is imaginary. Secondly, for $M = S^2$, we consider the imaginary time evolution for the hamiltonian that is half the square of the moment map for an S^1-action on S^2. In both cases, we study the evolution of the Kähler structure under the complex flow, and use graphical representations whenever possible.

This work is the result of a Master's dissertation, supervised by Prof. José M. Mourão.

References

[Don99] S. K. Donaldson. Symmetric spaces, Kähler geometry and Hamiltonian dynamics. In *Northern California Symplectic Geometry Seminar*, volume 196 of *Amer. Math. Soc. Transl. Ser. 2*, pages 13–33. Amer. Math. Soc., Providence, RI, 1999.

[MN15] José M. Mourão and João P. Nunes. On complexified analytic Hamiltonian flows and geodesics on the space of Kähler metrics. *Int. Math. Res. Not. IMRN*, (20):10624–10656, 2015.

Numerical continuation of solutions of neural field equations with oscillatory coupling functions

Weronika Wojtak, Flora Ferreira, Estela Bicho and Wolfram Erlhagen

Weronika Wojtak, Algoritmi Center and Center of Mathematics, University of Minho, Portugal

w.wojtak@dei.uminho.pt

Flora Ferreira, Algoritmi Center, University of Minho, Portugal

Estela Bicho, Algoritmi Center, University of Minho, Portugal

Wolfram Erlhagen, Center of Mathematics, University of Minho, Portugal

Neural field models, formalized by integro-differential equations, describe the large-scale spatio-temporal dynamics of neuronal populations [1]. They have been used in the past as a framework for modeling a wide range of brain functions, including multi-item working memory [2]. Neural field equations support spatially localized regions of high activity (or bumps) that are initially triggered by brief sensory inputs and subsequently become self-sustained by recurrent interactions within the neural population. We apply a special class of oscillatory coupling functions and analyze how the shape and spatial extension of multi-bump solutions change as the spatial ranges of excitation and inhibition within the field are varied [3]. More precisely, we use numerical continuation to find and follow solutions of neural field equations as the parameter controlling the distance between consecutive zeros of the coupling function is varied [4]. Important for a working memory application, we investigate how changes in this parameter affect the shape of bump solutions and therefore the maximum number of bumps that may exist in a given finite interval.

References

[1] S. Amari. Dynamics of pattern formation in lateral-inhibition type neural fields, *Biological Cybernetics,* 27 (2) pp. 77-87, 1977.

[2] C. R. Laing, W. C. Troy, B. Gutkin and G. B. Ermentrout. Multiple bumps in a neuronal model of working memory, *SIAM Journal on Applied Mathematics,* 63 (1), pp. 62-97, 2002.

[3] F. Ferreira, W. Erlhagen and E. Bicho. Multi-bump solutions in a neural field model with external inputs, *Physica D: Nonlinear Phenomena,* 326 pp. 32-51, 2016.

[4] C. R. Laing. Numerical Bifurcation Theory for High-Dimensional Neural Models, *The Journal of Mathematical Neuroscience,* 4 (1), pp. 13, 2014.